Intermediate Diagonal Tension Field Shear Beam Development for the Boeing SST

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This paper describes the comprehensive test and analysis program initiated for the development of titanium intermediate diagonal tension-field shear beam analysis and design methods necessary to support the Boeing SST. Existing semiempirical design and analysis procedures for aluminum beams have been extended for use with titanium beams. Detailed studies of the test results and current design methods show that major improvements of intermediate shear beam structural efficiency can be obtained through improved analytical procedures. A general theoretical analysis of post buckled stiffened plates, adaptable to the analysis of intermediate diagonal tension-field shear beams, was developed and is presented here. The theoretical formulation of this nonlinear problem was solved by the Raleigh-Ritz method. Improvements available after the theoretical program becomes fully operational should constitute a second generation of shear beam designs that will provide improved and more efficient structure for the SST and other new vehicles. Areas for improvements to shear beams that have evolved during the program are indicated.

Nomenclature

A = cross-sectional area of chord
A_s, A' = cross-sectional area of stiffener(s)
$C_{11}, C_{12}, C_{22}, C_{33}$ = elastic properties of isotropic web
c_k = distance locating the stiffeners
d = stiffener spacing
D = web bending rigidity
E = Young's modulus of the web or chord
E' = Young's modulus of stiffener
e_1 = displacement constant, effect of uniform bending moment on axial strain of beam
e_2 = displacement constant, effect of moment due to end shear load on axial strain of beam
e_3 = displacement constant, effect of moment on axial centerline deflection of beam
e_4 = displacement constant, effect of end shear load on centerline deflection of beam
F_c = stiffener forced crippling strength
F_{cy} = stiffener compression yield strength
f_s = applied shear stress in web
Fs_{all} = allowable web strength
F_{su} = web ultimate shear strength
F_{tu} = web tension ultimate strength
F_{ty} = web tension yield strength
h = total height of beam
h_c = distance between the upper and lower chord centroids
I_c, I_{zz} = moment of inertia of chord about z axis
$I_{xx'}$ = moment of inertia of stiffener about x axis
I_{xz}' = product of inertia of stiffener in xz plane
I_{zz}' = moment of inertia of stiffener about z axis
k = diagonal tension factor
K_s = plate buckling coefficient in shear
K_{sc} = stiffener stress correction factor
ℓ = length of the beam
m_1 = row indice on u-displacement coefficient
n_1 = column indice on u-displacement coefficient
P = shear load on the beam
P_{cr} = shear load on beam producing initial buckling
p_1 = row indice on v-displacement coefficient
q_1 = column indice on v-displacement coefficient
r_1 = row indice on w-displacement coefficient
s_1 = column indice on w-displacement coefficient
t = thickness of web
t_s = thickness of stiffener flange attached to web
u = displacement in x direction
$u_{m_1n_1}$ = u-displacement coefficient
U = total potential energy
v = displacement in y direction
V_b = bending strain energy of web
V_c = strain energy of chords
V_m = inplane strain energy of web
V_s = strain energy of stiffeners
$v_{p_1q_1}$ = v-displacement coefficient
v_{oq_1} = v-displacement coefficient, $p_1 = 0$
w = displacement in z direction
w_{max} = maximum out-of-plane deflection of web
W = work of external shear force
$w_{r_1s_1}$ = w-displacement coefficient
x,y,z = coordinate axes
$\bar{\gamma}$ = displacement constant, shearing stress at root of beam ($x = 0$)
δ = variational operator
μ = Poisson's ratio

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Introduction

THE general problem of intermediate tension field beams has been treated analytically by Wagner[1] (1929) and Wagner[2] (1935), Koiter[3] (1944), Denke[4] (1944), Levy[5] (1945) and Djubek[6] (1966) and empirically by Lipp[7] (1939), Kuhn[8] (1952) and Rockey[9] (1957). The analytical theory for incomplete diagonal tension or intermediate shear beams is very complex. Prior investigators have introduced various simplifying assumptions in order to make the problem mathematically tractable. In doing so, they have limited the range and type of beams that can be analyzed, thereby reducing the usefulness of their analysis procedures. The procedures resulting from theoretical approaches also have not lent themselves to simple design methods easily used by the design engineer. As a result, the material dependent empirical procedures developed by Lipp,[7] and Kuhn[8] are the most widely used in the aircraft industry. These two methods were developed for 2024-T3 and 7075-T6 aluminum.

It was recognized early in the Boeing Supersonic Transport program that the present empirical design procedures for intermediate shear beams could not be readily extended to other materials such as titanium.[10] After a thorough literature search, it became apparent that considerable scatter exists between predictions and tests. The existing theoretical analysis methods, for the most part carried out before the age of the computer, were not sufficiently accurate. The SST requires development of structure with higher efficiencies due to performance requirements and associated economic implications.[11] A comprehensive program involving both test and mathematical analysis was necessary to develop new stress and strength analysis methods for titanium intermediate shear beams. The tests have been accomplished.[12] Development of a computer program providing a nonlinear post buckled analysis of a shear beam has been completed and documented.[13] However, reduced SST funding has dictated the termination of the shear beam development program without combining the results of those two phases of the effort. Those results need to be combined in order to develop the improved strength prediction method and the new generation of shear beam designs as anticipated when the program was initiated.

The purpose of this paper is to show that major improvements can be made in the predictions of strength of intermediate shear beams designed by existing methods, that the development of a more general theoretical analytical procedure for analysis of stiffened plates may be applied to the intermediate shear beams, and that improved families of beam designs with increased efficiency will become available when the analytical tools are adequately developed and employed.

Study of the Kuhn Method

An examination of the Kuhn method presented in NACA TN 2661 and 2662[8] revealed large discrepancies exist between some of the tests and the strengths predicted by the method. This fact is understandable since the treatment in NACA TN 2661 is empirical. The various correction coefficients used are taken from the lower boundary of the scatter band of approximately two hundred beam tests. An example of this conservatism in the method is shown in Fig. 1. This figure provides a plot of a nondimensional web tear strength (f_s/F_{tu}) versus a nondimensionalized web buckling coefficient. The indicated test points include 2024-T3 and 7075-T6 data from NACA TN 2662. The plotted data was analyzed as in NACA TN 2661 without regard to percent stiffening and other parameters. The figure shows a broad scatter band in the test data with the allowable strength curve established at the bottom of the scatter band of that data.

This conservative approach is necessarily followed throughout in the development of the Kuhn analysis method for intermediate shear beams. As a result, beams designed per NACA TN 2661 have proved to be reliable. Thus, the method has been used extensively in the design of most present-day aircraft. The fact that the tools that were presented in NACA TN 2661 were developed before the general use of major computer programs and have served so well for so many years must be considered as remarkable.

SST Shear Beam Developmental Program Requirements

In order to satisfy the SST requirement for higher structural efficiencies, efforts were made to obtain consistent, smaller margins between tests and predictions for intermediate shear beams. This necessitated considering various geometric parameters not used previously, classifying types of loading and stiffeners, and improving representation of material properties. Since the SST primary structural material is titanium, another major task was extending the analysis method to cover this material. Careful control of specimen geometric parameters, material properties, and testing procedures were exercised in order that test data scatter could be reduced. The test program was set up to determine the effects on ultimate strength and other characteristics of a titanium beam due to variations of web thickness, stiffener spacing, panel depth, stiffener type and size, chord stiffness, and loading (combined shear and moment, combined shear and axial load).

Testing

After reviewing prior shear panel test systems, it was decided to use a system that permitted testing of fully cantilevered beams while controlling both axial load and bending moment.[12] A typical shear beam test setup is shown in Fig. 2. Ratay[14] independently selected a similar test system.

A grid-shadow Moire technique was developed and used to observe the growth of buckle patterns in the web.[15] Extensive use was also made of strain gages (Figs. 3 and 4).

The initial test series of six aluminum and four titanium cantilevered beams were loaded in shear, in shear plus axial tension, and in shear plus axial compression. These initial tests have provided orderly trends with failures initiating in the stiffeners and webs. Beam strengths were decreased from 4% to 12% by adding axial compression load approximately equal to the shear load on the beam, or increased by approximately 7% by adding axial tension load in the range tested. Table 1 shows the various geometric properties and the test results for each beam.

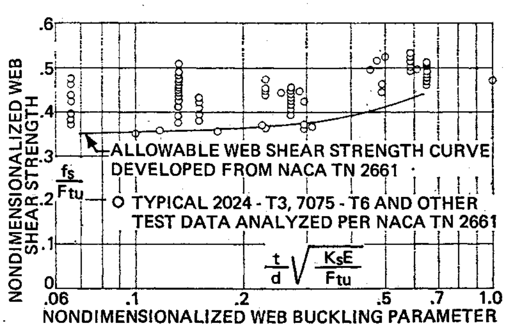

Fig. 1 Intermediate shear beam test results.

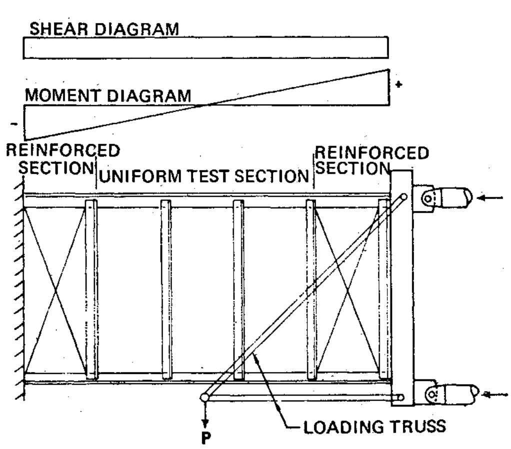

Fig. 2 Typical shear beam test.

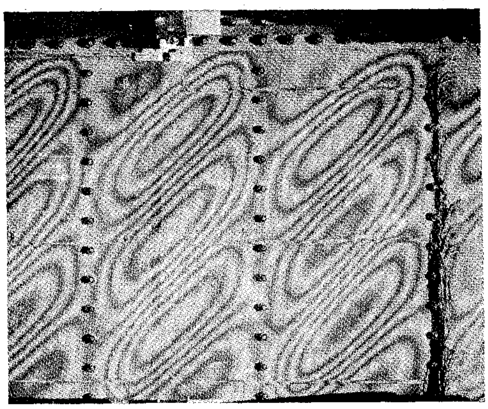

Fig. 3 Shear web viewed with Moiré grid at low load.

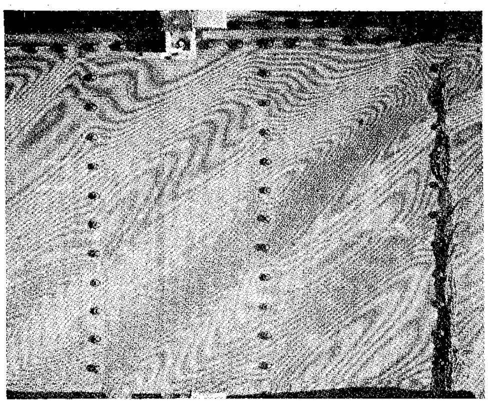

Fig. 4 Shear web viewed with Moiré grid at high load.

Using the methods indicated in NACA TN 2661, without any attempts to account for combined loading, the test results ranged from 22% to 88% above the predicted strengths. For the titanium beams, linear extrapolations of the aluminum web and stiffener strength prediction methods were used. The results for the second test series are shown in Table 2. The first two tests (#12, 13) are duplicates of specimens in the first series (#9, 7) except they are 5 instead of 3 bay beams, (Figs. 5 and 6).

TABLE 1 INTERMEDIATE SHEAR BEAMS—TEST SERIES I [a]

SPEC. NO.	MAT'L	WEB (t)	STIFFENER AREA RATIO A_S/dt	FAILURE LOAD (kips)	TEST PREDICT. b	END LOAD (kips)
2	ALUM.	.071	.450	57.7	1.27	
3	TITAN.	.050	.360	60.0	1.52	
4	ALUM.	.071	.450	55.3	1.22	50 COMP. (4% STRENGTH DECREASE DUE TO COMPRESSION)
5	TITAN.	.050	.370	56.5	1.33	50 COMP. (6% STRENGTH DECREASE DUE TO COMPRESSION)
6	ALUM.	.071	.235	49.7	1.43	
7	TITAN.	.043	.270	54.0	1.88	
8	ALUM.	.071	.235	43.7	1.26	50 COMP. (12% STRENGTH DECREASE DUE TO COMPRESSION)
9	ALUM.	.071	.235	46.0	1.33	c (7% STRENGTH DECREASE DUE TO COMBINED MOMENT AND SHEAR)
10	ALUM.	.071	.235	53.2	1.54	50 TENS. (7% STRENGTH INCREASE DUE TO TENSION)
11	TITAN.	.045	.260	51.0	1.72	50 COMP. (6% STRENGTH DECREASE DUE TO COMPRESSION)
				AVE.	1.45	

a ALL TEST SPECIMENS: 3 BAYS, d = 9", DEPTH = 24"
b BASED ON EXTRAPOLATION OF NACA TN 2661
c LOADED IN SHEAR AT END OF BEAM (COMBINED SHEAR PLUS MOMENT)

TABLE 2 INTERMEDIATE SHEAR BEAMS—TEST SERIES II [a]

SPEC. NO.	MAT'L	WEB (t)	STIFFENER AREA RATIO A_S/dt	FAILURE LOAD (kips)	TEST PREDICT. b		DESCRIPTION
12	ALUM.	.071	.235	38.0	1.10		
13	TITAN.	.042	.290	41.3	1.36		
14		.036	.885	37.2	1.67	TYPICAL SST FUSELAGE PANEL DESIGNS	
15		.049	1.02	53.8	1.44		SIMILAR TO 18 EXCEPT FOR .013" PAD ON WEB AT WEB-TO-STIFFENER RIVET LINES.
16A		.041	.560	37.2	1.29		
16B		.041	.570	39.7	1.35		SAME AS 16A EXCEPT FOR .010" PAD ON WEB AT WEB-TO-STIFFENER RIVET LINES.
17		.032	.600	37.0	1.93		
18	TITAN.	.053	.885	53.0	1.40		
				AVE.	1.44		

a : (1) SPECIMENS 12 & 13: 5 BAYS, d = 9", DEPTH = 24"
(2) ALL OTHER BEAMS: 7 BAYS, d = 4.5", DEPTH = 18", EXCEPT NO. 17 WHICH HAS 5 BAYS WITH d = 6".
b BASED ON EXTRAPOLATION OF NACA TN 2661

Comparison of the results of like tests led to the conclusion that beam length effect must be taken into account in predicting strengths. The remaining six tests represent typical SST fuselage shear panel designs. Reduced shear head countersunk titanium rivets were used to attach the stiffeners to the webs, and premature failure of some of the panels occurred due to rivet tension pull-out. From analysis of stiffener and web strain gage data, it appears that the ultimate strengths of the beams in these cases were nearly reached and failure would have occurred at loads close to the observed loads had the rivets not failed. The test results range from 10% to 93% above the strength predicted by the methods indicated by Kuhn extrapolated to account for the properties of titanium.

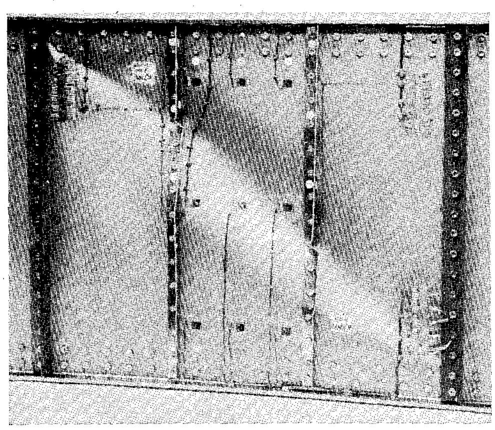

Fig. 5 Failed 3-bay diagonal tension shear beam.

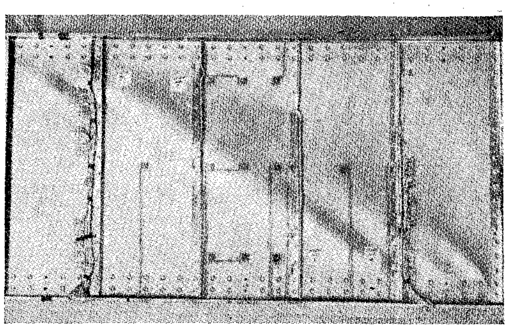

Fig. 6 Failed 5-bay diagonal tension shear beam.

Discussion of Test and Analytical Correlations

Modifications to the analysis method were made to reduce the large discrepancies that existed between predicted strengths and the strengths obtained from the tests. Linear extrapolations of web and stiffener strengths were replaced by empirical equations derived from the test data. The following equation developed by Tsongas and Ratay[16] was used for web strengths:

$$F_{s_{all}} = 0.90F_{ty}\left[1+\frac{1}{2}\left\{\frac{F_{tu}}{F_{ty}}-1\right\}^2\right]\left[\frac{1}{2}+(1-k)^3\left\{\frac{F_{su}}{F_{tu}}-\frac{1}{2}\right\}\right] \qquad (1)$$

The following material dependent equation was used for the stiffener forced crippling strengths

$$F_c = 0.00058E\left[\frac{F_{cy}}{1000}\right]^{0.4} k^{2/3}\left[\frac{t_s}{t}\right]^{1/3} \qquad (2)$$

The strength predictions resulting from these two equations have proven to be consistently conservative and have helped to reduce the scatter between tested and predicted strengths. Another improvement to the Kuhn method was the addition of a beam length effect reduction coefficient which reduced the calculated applied stiffener stress.

In the present Kuhn method, the applied stiffener stress is calculated based on the assumption that the stiffener supports one panel length of diagonal tension. In actuality, the chord acts as a continuous beam over several supports thereby reducing axial stresses in the stiffeners between the stiff end bay members. The Kuhn method recognizes the effect of beam length on beam strength indirectly through the "portal frame" correction to web stresses, but it does not treat the direct effect upon the stiffener stresses. Figure 7 gives the stiffener stress correction factor as a function of number of bays in the beam and chord beaming rigidity which can be applied to correct for applied stiffener stresses. These curves are based on continuous beam theory.

Using the preceding modification to the Kuhn method, the differences between test and predicted strengths have been reduced considerably. Table 3 gives the ratios of test over prediction for both the extrapolated Kuhn and the modified Kuhn methods for all the tests. Using the modified Kuhn method reduces the average over-strength ratio from 1.45 to 1.18. This is a significant improvement. Note that no effort was made to sort out the effects of combined loading in the modified analysis method. Those effects are small compared to the data scatter.

Although comparisons in Table 3 show considerable improvements, the (remaining scatter) results show that there are major improvements yet to be made. Those improvements must be made in order to fully satisfy the requirements for efficient structure. Requirements for optimizing shear beam designs and for the handling of combined loads further necessitate improvement in analysis methods. Since the number of parameters affecting the strength of shear beams are so numerous, it is an economic impossibility to conduct the amount of testing that would be required to develop a strictly empirical efficient, multipurpose, general analysis method. For this reason, a theoretical approach to the nonlinear analysis of stiffened plates applicable to the analysis of intermediate shear beams was initiated along with the above analysis and test work. It is hoped that the general theoretical approach to the problem, along with test data, can be used to develop general analysis procedures adaptable to intermediate shear beams of any material with combined loading and with various geometric properties.

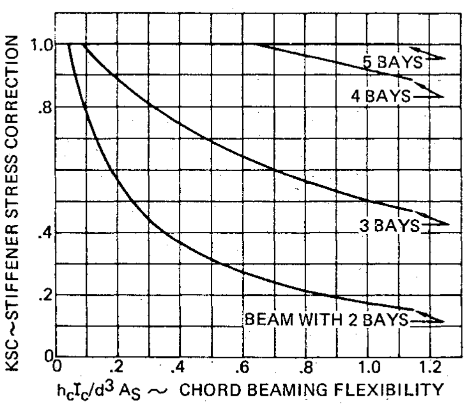

Fig. 7 Stiffener stress correction factor due to chord beaming.

Table 3 Comparison of prediction methods

Test No.	Test / Pred. NACA TN 2661 Extrap.	Test / Pred. NACA TN 2661 Modified
2	1.27	1.18
3	1.52	1.10
4	1.22	1.13
5	1.33	1.03
6	1.43	1.24
7	1.88	1.40
8	1.26	1.09
9	1.33	1.15
10	1.54	1.33
11	1.72	1.31
12	1.10	1.09
13	1.36	1.09
14	1.67	1.29
15	1.29	1.04
16a	1.29	1.04
16b	1.35	1.06
17	1.93	1.47
18	1.40	1.10
Average	1.45	1.18

Theoretical Development (Sherrer Option Program)

A solution is presented to the nonlinear problem of the deformation of thin rectangular plates which are stiffened as follows: by elastically compressible stiffeners along the vertical edges, by elastically compressible chords flexible in the plane of the plate along the horizontal edges, and by intermediate equally spaced vertical stiffeners that are elastically compressible with in-plane, out-of-plane and nonsymmetric bending flexibilities. The web-plate is assumed to be simply-supported along its boundary and "in-plane" cantilever beam type shear and bending deformation is allowed.

Formulation of the Problem

Consider a rectangular webplate of dimensions ℓ and h acting as a cantilever beam, supported at the left, stiffened along its edges and by intermediate vertical stiffeners, and subject to a shear load as indicated in Fig. 8.

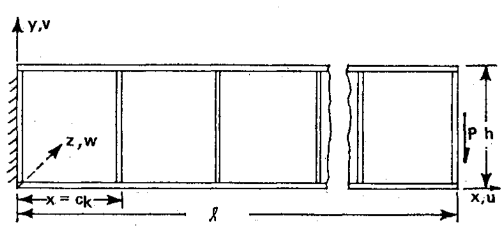

Fig. 8 Reinforced rectangular web.

Under loading, the beam will show both shear and bending distortions. The horizontal chords will shorten or lengthen axially due to bending loads and will deflect under the action of the post buckling membrane forces in the xy plane. The internal stiffeners will shorten axially and deflect inplane and out-of-plane under the action of the postbuckling membrane forces. The edge vertical stiffeners are allowed to shorten axially under the action of the membrane forces.

In the direction perpendicular to the plane of the webplate, the edge stiffeners are considered to have sufficient flexural rigidity so that the condition of zero deflection of the webplate along its boundary is satisfied. The solution is restricted to consideration of the case of a plate simple-supported along its boundary, i.e., assuming that the edge stiffeners do not prevent the webplate from rotating along the edges. It is further assumed that the webplate has no initial curvature.

The theoretical formulation of the nonlinear problem is carried out using a potential energy formulation. In order to get coupling of the inplane and out-of-plane deformations (and hence the "post buckling" effect of the thin web structural system), the nonlinear strain-displacement equations of elasticity must be used when the inplane strain energy of the web is written. The total potential energy expression for the system is written as follows:

$$U = V_B + V_M + V_S + V_C - W \tag{3}$$

Using the classical strain energy expressions for beams and plates from the literature,[17] the total potential energy expression for the structural system may be written as follows:

$$\begin{aligned}
U = {} & \frac{t^3}{24}\int_0^h\int_0^{\ell}\left[C_{11}\left\{\frac{\partial^2 w}{\partial x^2}\right\}^2 + C_{22}\left\{\frac{\partial^2 w}{\partial y^2}\right\}^2 + 4C_{33}\left\{\frac{\partial^2 w}{\partial x \partial y}\right\}^2 + \right. \\
& \left. 2C_{12}\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right\}\right] dxdy + \frac{1}{2}t\int_0^h\int_0^{\ell}\Big\}C_{11}\left[\left\{\frac{\partial u}{\partial x}\right\}^2 + \right. \\
& \left. \frac{\partial u}{\partial x}\left\{\frac{\partial w}{\partial x}\right\}^2 + \frac{1}{4}\left\{\frac{\partial w}{\partial x}\right\}^4\right] + C_{22}\left[\left\{\frac{\partial v}{\partial y}\right\}^2 + \frac{\partial v}{\partial y}\left\{\frac{\partial w}{\partial y}\right\}^2 + \right. \\
& \left. \frac{1}{4}\left\{\frac{\partial w}{\partial y}\right\}^4\right] + C_{33}\left[\left\{\frac{\partial u}{\partial y}\right\}^2 + \left\{\frac{\partial v}{\partial x}\right\}^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + 2\frac{\partial u}{\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \right. \\
& \left. 2\frac{\partial v}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \left\{\frac{\partial w}{\partial x}\right\}^2\left\{\frac{\partial w}{\partial y}\right\}^2\right] + 2C_{12}\left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{1}{2}\frac{\partial u}{\partial x}\left\{\frac{\partial w}{\partial y}\right\}^2 + \right. \\
& \left. \frac{1}{2}\frac{\partial v}{\partial y}\left\{\frac{\partial w}{\partial x}\right\}^2 + \frac{1}{4}\left\{\frac{\partial w}{\partial x}\right\}^2\left\{\frac{\partial w}{\partial y}\right\}^2\right]\Big\} dxdy + \frac{A'E'}{2}\int_0^h\left\{\frac{\partial v}{\partial y}\right\}^2_{x=ck} dy + \\
& \frac{E'I_{zz}'}{2}\int_0^h\left\{\frac{\partial^2 u}{\partial y^2}\right\}^2_{x=ck} dy + \frac{E'I_{xx}'}{2}\int_0^h\left\{\frac{\partial^2 w}{\partial y^2}\right\}_{x=ck} dy + \\
& \frac{E'I_{xz}'}{2}\int_0^h\left\{\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 w}{\partial y^2}\right\}_{x=ck} dy + \frac{AE}{2}\int_0^{\ell}\left\{\frac{\partial u}{\partial x}\right\}^2_{y=o,h} dx + \\
& \frac{EI_{zz}}{2}\int_0^{\ell}\left\{\frac{\partial^2 v}{\partial x^2}\right\}^2_{y=o,h} - Pv\Big|_{x=\ell}
\end{aligned} \tag{4}$$

By including the various energies above in the post buckling analysis of a stiffened plate, all the effects considered by previous investigators are taken into account plus several additional effects such as: 1) combined shear and bending; 2) unsymmetric stiffeners; 3) multibay beams; 4) out-of-plane deflection of the intermediate vertical stiffeners.

This formulation has been generalized further in Boeing Document AS 2794[13] to include the torsional and warping rigidities of the chords and stiffeners.

Method of Solution

The method of solution used is the Raleigh-Ritz technique for minimization of the total potential energy with assumed in-plane and out-of-plane displacement functions that satisfy the boundary conditions of the stiffened plate. The solution to the problem is obtained by, taking the first variation of the total energy, substituting the assumed displacement expression functions into this variational expression, and then integrating. This leads to a set of simultaneous nonlinear equations in terms of the undetermined coefficients of the assumed displacement functions. By assuming displacement functions in terms of summation type series, there is flexibility in choosing the appropriate coefficients required to give an accurate solution to the problem. Since the formulation is worked out in terms of summations, the computer automatically does the necessary expansions and the number of terms or coefficients used in the displacement functions can be any number deemed necessary to get a good solution. Solution time and storage capacity are the limiting conditions on the number of parameters.

The displacement functions assumed are taken in the form

$$u = e_1 x\left(y - \frac{h}{2}\right) + e_2\frac{x^2}{2}\left(y - \frac{h}{2}\right) + \sum_{m_1}\sum_{n_1} u_{m_1 h_1}\sin\frac{m_1\pi x}{\ell}\sin\frac{n_1\pi y}{h} \tag{5}$$

$$v = \bar{\gamma}x + e_3x^2 + e_4x^3 + \sum_{q_1} v_{oq_1}\cos\frac{q_1\pi y}{h} + \sum_{p_1}\sum_{q_1} v_{p_1q_1}\cos\frac{q_1\pi x}{\ell}\cos\frac{q_1\pi y}{h} \tag{6}$$

$$w = \sum_{r_1}\sum_{s_1} w_{r_1s_1}\sin\frac{r_1\pi x}{\ell}\sin\frac{s_1\pi y}{h} \tag{7}$$

where e_1, e_2, e_3, e_4, $\bar{\gamma}$, $u_{m_1h_1}$, v_{oq_1}, $v_{p_1q_1}$ and $w_{r_1s_1}$ are undetermined displacement parameters. In Eqs. (5) and (6), which give the inplane displacements of the stiffened plate, the polynomial terms are chosen to take care of the elementary beam deflections due to bending and shear. The Fourier terms correct the errors introduced by the polynomial assumptions. Equation (7) for the out-of-plane deflections of the system satisfies the boundary conditions of the beam web.

These displacement functions satisfy the beam boundary conditions and allow for the following beam behavior: 1) shear and bending deformations of the beam, inplane bending, out-of-plane bending, and axial deformation of the internal stiffeners; 3) inplane bending and axial deformation of the chords; 4) axial shortening of the vertical edge stiffeners.

The displacement equations and their variations are then substituted into the first variation of Eq. (4) and the results integrated. This leads to an expression for the first variation of the total potential energy in terms of the deformation parameters and establishes the equilibrium of the beam system as follows:

$$\delta U(\delta e_1, \delta e_1, \delta e_2, \delta e_2, \ldots, \text{etc.}, \ldots) = 0 \tag{8}$$

The coefficients for δe_1, δe_2, etc. are collected and each set to zero, respectively. A set of nonlinear simultaneous equations in terms of the undetermined deformation parameters and the load P will result.

The equations are then written with the linear terms on the left-hand side (LHS) of the equations and with the loading and nonlinear terms on the right-hand side (RHS). If the nonlinear terms on the RHS of these equations are set to zero, the remaining set of linear equations give the solution to the linear, small deflection beam problem. Also, the buckling problem of stiffened plates can easily be deduced from the results.

Numerical Solution

First, the appropriate number of terms for the displacement functions considered necessary for a good solution are selected. Then through the summation process, the equations derived in the previous section are expanded to give an appropriate number of equations agreeing with the number of deflection terms selected. The displacement coefficients (e_1, e_2, e_3, e_4, $\bar{\gamma}$, v_{01}, u_{11}, w_{11}, ... etc.) are now obtained by the solution of the set of nonlinear simultaneous equations. These equations are solved using a Gauss-Siedel iteration procedure. This procedure for the solution of the system of nonlinear equations is outlined below.

1) The set of equations are solved for the displacement parameters. Note that the nonlinear system of equations is solved in terms of the w_{11} displacement parameter and not in terms of the loading as would be done conventionally. This is done in order to get better convergent solutions to the problem and is consistent with what was done previously in the literature (see NACA TN 962).[5] Further, it is to be noted that the problem should be solved in terms of the most dominant displacement parameter in order to get good solutions.

2) For the first iteration, values of w_{22}/t, w_{13}/t, w_{33}/t, P and $\bar{\gamma}$ are estimated corresponding to a given value of w_{11}/t. Then, values of the displacement coefficients and load P are calculated.

3) Improved values of the displacement coefficients and load P are obtained using the above equations with the values of the unknown parameters calculated from the first iteration. It is usually necessary to use under-relaxation procedures to obtain convergence.

4) This process is repeated until there is convergence for each displacement coefficient and load P.

Once the system of nonlinear equations converges, the deflection and stress state of the beam is easily calculated.

Numerical Results

Numerical results are obtained as outlined below:

1. *Single bay stiffened plate*

The deformations of a single bay stiffened plate (vertical edge stiffeners that allow only axial shortening and horizontal edge chords that allow inplane bending) have been computed choosing the following coefficients of the deflection functions:

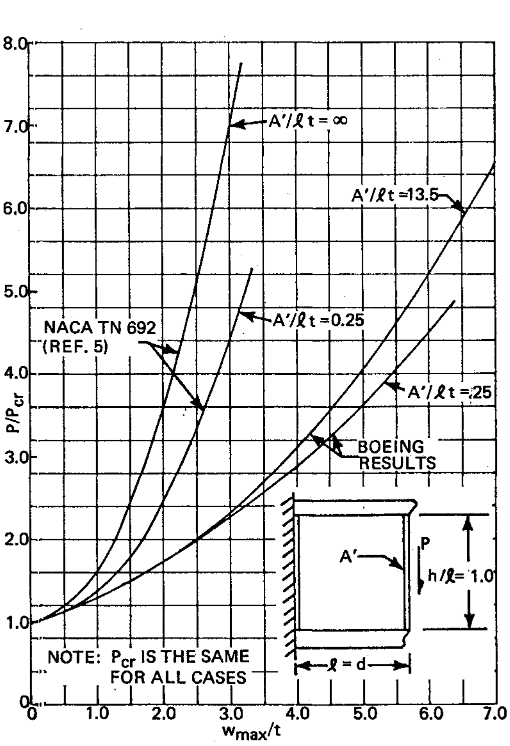

Fig. 9 Effect of stiffener area single bay beam.

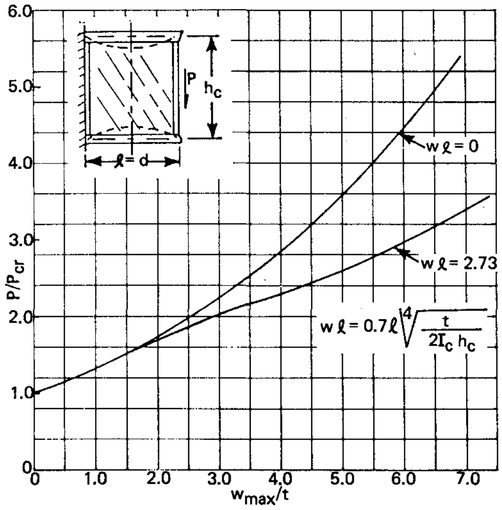

Fig. 10 Effect of chord flexibility single bay beam.

$\bar{\gamma}$, v_{01}, v_{12}, v_{14}, v_{16}, v_{18}, w_{11}, w_{22}, w_{13}, w_{33}. For a given beam geometry, the coefficient w_{11} was chosen with values $0.1t$, $0.5t$, $1.0t$, etc. Each solution is considered the first iteration of the next solution. Good convergence of the nonlinear equations have been obtained in the low post-buckling range

$$(P \leq 5p_{cr} \quad \text{and} \quad w_{max}/t \leq 5)$$

For this single bay beam, the influence of stiffener area and chord flexibility has been investigated. Figure 9 shows the effect of stiffener area on the maximum deflection of a square plate loaded in shear. For each case shown, the chord geometry is not changed. Figure 10 shows the effect of inplane chord flexibility on the maximum deflection of a rectangular plate loaded in shear. Stiffeners remained unchanged for each case.

2. *Two bay stiffened plate*

The deformation of a two bay stiffened plate have been computed choosing the following coefficients of the deflection functions; λ, v_{01}, v_{12}, v_{14}, v_{16}, v_{18}, $w_{r_1 s_1}$ ($r_1 = 1$ ---, 6; $s_1 = 1$, --, 4 with $r_1 + s_1 =$ even terms only).

Figure 11 shows the influence of the out-of-plane flexibility of the interior stiffener on the buckling load of the plate and the maximum deflection of the plate.

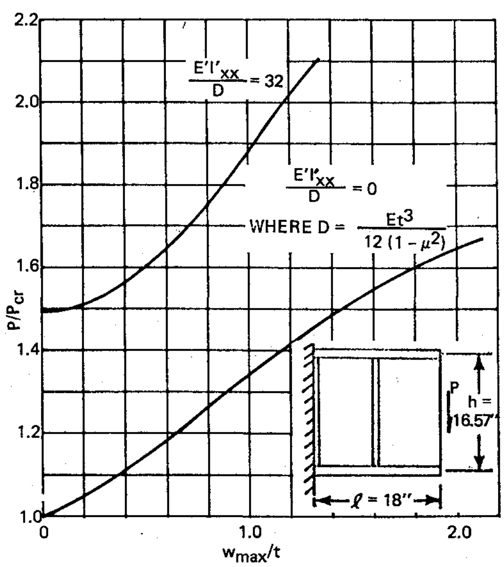

Fig. 11 Effect of stiffener out-of-plane flexibility—two bay beam.

Conclusions

Current analysis and tests indicate that major improvements are available in the structural efficiency of intermediate tension field shear beams. Detail studies of current analysis methods for these types of beams show them to be conservative, limited in application (material dependent for example), and in need of improvement in providing more efficient, optimized designs. To extend these present semiempirical analysis methods to the parameter ranges, loadings and materials required to provide for optimized designs of shear beams through additional testing alone is an economic impossibility. For this reason, a general theoretical approach to the post buckling analysis of stiffened plates that can be adapted to the analysis of intermediate diagonal tension field beams, such as that discussed here, must be used as the primary tool for further sensitivity investigations and design improvements.

The improvements available, after the theoretical program becomes fully operational and is used to handle the most general cases (only results of limited problems are shown here), should constitute a second generation of shear beam designs that will provide improved and more efficient structures. Some of the areas of interest that have evolved from the present work are as follows: 1) Alternate light and heavy stiffeners to take advantage of the length effect; 2) skewed stiffeners; 3) chem-milled web designs (pads at the web to chord and stiffener attachments); 4) secondary stiffeners parallel to the chords to create deep beam chords; 5) integral web-stiffener and welded designs; 6) orthotropic webs.

The general theoretical program should permit these concepts to be simulated and evaluated, evolving combinations of these features most advantageous to the design requirements for a particular aircraft.

Although this treatment deals only with static strength, it is recognized that the design of shear beams and other structure is strongly influenced by fatigue considerations. The improvements in design, static strength, stress analysis, and deflection analysis as discussed above must be accomplished first in order to permit development of a meaningful fatigue analysis of intermediate shear beams.

Hopefully these efforts, initiated to support the Boeing SST design, will be of value in the design of other systems which also require high structural efficiency.

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